

TM-1546

# Third Order TRANSPORT with MAD Input

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September 20, 1988





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#### I. MAD Notation

## I-A. Specification of Elements

The MAD (Methodical Accelerator Design) notation differs from that of traditional TRANSPORT in three basic ways. They are:

- (1) Elements are designated by mnemonic names rather than numerical type codes.
- (2) The element parameters are designated by keywords rather than by position.
- (3) Elements are specified first, then assembled into an optical system by additional program instructions.

This last difference occurs because accelerators typically have a high degree of repetition, whereas secondary beam lines often have very little. Either MAD or traditional TRANSPORT notation can be used in any of these three ways. The choice of a particular notation for one case does not dictate that choice for another of the three items. For example, elements specified in traditional TRANSPORT notation may be assembled into beam lines using MAD instructions. Any element can be specified using either notation. In addition there are many hybrid notations for the specification of elements which often prove useful.

Let us begin by discussing the first two of the three items. The third we will describe in a later section on beam line definitions.

In traditional TRANSPORT notation, the specification of a bending magnets might appear as

4. 10. 20. 0.5 'BM1';

The number 4. is the type code indicating a bending magnet. The following numbers indicate that the length is 10 meters, the magnetic field 20 kilogauss, and the normalized field gradient (a pure number) is 0.5. The label 'BM1' is the proper name of this particular bending magnet.

The type code may be replaced by a more easy-to-read mnemonic to give the following appearance

BEND 10. 20. 0.5 'BM1';

The label can also be identified by following it with a colon, such as

BM1: BEND 10. 20. 0.5;

Either form of label may be placed anywhere among the numbers.

In keyword notation the variables are identified by name. A specification of the same bending magnet in terms of the same variables might be

BEND 'BM1' 
$$L = 10.$$
,  $B = 20.$ ,  $N = 0.5$ ;

Notice that commas are needed to separate keyword entries. Commas are not needed for positional parameters. The order of the parameters is not significant in keyword notation. The above specification is equivalent to

BEND 
$$B = 20., L = 10., N = 0.5$$
 'BM1';

The label in this case need not be separated from the previous parameter specification by a comma. The opening single quote serves to identify the label. If a label in the same position is identified by a colon, then it must be preceded by a comma.

Notations may also be mixed within a single element. One may begin with positional parameters, then switch to keyword parameters. Our bending magnet might then look like

```
BEND 10. 20. N = 0.5, BM1: ;
```

On a particular element, once one has switched to keyword parameters, one may not switch back to positional parameters. The element is terminated by a semicolon. The next element may then be written in any manner desired.

The mixed mode of specifying parameters is especially useful for parameters which need not always be specified. If the magnetic field of the bending magnet has a quadratic dependence (significant only in a second- or higher-order calculation), it might be written as:

```
BEND 10. 20. 0.5 \text{ EPS} = 0.01 \text{ 'BM1'};
```

The bending magnet may also be specified in terms of a different set of parameters. Instead of the length, magnetic field, and normalized derivative n, we might use the length, bend angle, and alternate normalized derivative K1. For a 600 GeV beam, our bend magnet might be specified as

```
BEND L = 10., ANGLE = 10., K1 = 0.5E-6 'BM1';
```

It is this last set of variables which is used to specify a bending magnet in the MAD program itself. It has the advantage that it does not require the beam momentum to calculate the transfer matrix.

All of these specifications, along with several others, can be understood by the TRANSPORT program. In the following pages we describe each element and the various keywords that may be used to specify the values of its parameters. We start with a list of the mnemonics which may be used instead of the traditional numerical type codes. The names used are the same as occur in the printout of the TRANSPORT program. These names are all the same as used in the MAD program. Some of the names occuring in the printout have been changed slightly from older versions of TRANSPORT.

## I-B. List of Type Codes and Mnemonics

Type		
Code	Mnemonic	Element
1.	BEAM	Input phase space and momentum
2.	ROTA (TION)	Fringing fields and pole-face rotations for bending magnets
3.	DRIF(T)	Drift space a field free region
4.	BEND \	Wedge bending magnet (without fringing fields)
5.	QUAD (RUPO)	Quadrupole magnet
6.	ŬPDA (TE)	Transfer matrix update
7.	CENT (ROÍD)	Shift in the beam centroid
8.	ALIG(N)	Magnet alignment tolerances
9.	REPE (AŤ)	Repetition of a segment of a beam line
10.	FIT `	Fitting constraint
11.	ACCE (LERA)	Travelling wave linear accelerator
<b>12</b> .	CORR (ELAT)	Beam rotated ellipse
13.	PRIN(T)	Output print control instructions
14.	MATR (IX)	Arbitrary matrix input
15.	UNIT(S)	Input-output units
16.	SPEC(IAL)	Special input parameters
<b>17</b> .	ORDE (R)	Zeroeth, first, second, or third order
18.	SEXT (UPOL)	Sextupole magnet
19.	SOLE (NOID)	Solenoid magnet
20.	SROT	Coordinate Rotation
<b>24</b> .	SECT (ION)	Define section of beam line
	OCTU (POLE)	Octupole magnet
	RAND (OM)	Random errors on physical parameters
	ETA	Accelerator eta function
	RBEN(D)	Rectangular bending magnet (with fringing fields)
	SBEN(D)	Sector bending magnet (with fringing fields)
	PARA (METE)	Parameter value
	MARK (ER)	Position marker
	STOR (E)	Storage of matrix element values
	PLAS (MALE)	Plasma (lithium) lens
	HKIC(K)	Horizontal steering magnet
	VKIC(K)	Vertical steering magnet

The first four letters of a mnemonic are necessary for its specification. More may be used for clarity if desired. For example, the mnemonic SECTION may provide greater clarity than just SECT, which, at first glance, might be taken to mean sector magnet. If more than four characters are used, they must be correct. A mnemonic of SECTOIN will not be recognized. The same restrictions apply to the keywords. The use of parentheses in this report is to show the characters which are non essential.

#### II. Preliminary Specifications.

#### II-A. Indicator Card

The single integer on the indicator card indicates whether the following data represent a new problem or modifications to a previously specified problem. An indicator of 0 indicates a new problem. An indicator of 1 indicates modifications to a previously specified problem.

As TRANSPORT developed various other options were introduced, allowing control over various stages of the printing. The keyword format of MAD allows greater flexibility with less confusion. The previously defined variations on the indicator card have the same effect as before, but are now unnecessary. The various print controls, including several new ones, can now all be specified via keywords. The keywords are placed on the indicator card, following the indicator number. If there is more than one, they must be separated by commas. The complete listing of such keywords is

#### Keyword Meaning

NOLIST The initial listing is suppressed.

REFORMAT The initial listing is reformatted. The output from this initial listing now resembles the uniformly spaced sequence of numbers used in older versions of TRANSPORT. The default is now to simply reprint the input data. The REFORMAT option should not be used with keyword input as this option assumed positional parameter input and the use of traditional TRANSPORT variables.

NOBEFORE The printing from the run through the beam line before fitting is suppressed. If there is no fitting to be done, this option will have no effect.

NOPRINT The printing from the run through the beam line both before and after fitting is suppressed. The printing during fitting remains. The output will still contain the sequence of chi-squared values and the changes in the varied parameters.

NOSOLVE The printing from the run through the beam line before, during, and after fitting is suppressed. The fitting is itself suppressed. The NOSOLVE option is useful on a 1 indicator card after the fitting has been completed in previous steps. The initial deck listing using the REFORMAT command can sometimes be used as data for subsequent TRANSPORT runs.

If the options NOLIST and NOSOLVE are both used, there is no output. For example, if one wanted only the listing printed after fitting is complete, then the indicator card might read

O NOLIST, NOBEFORE

The use of these indicator options along with the two commands

```
PRINT ELEMENTS, OFF; PRINT ONLY;
```

allows for great flexibility in determining the output to be printed. The first command suppresses the printing of the physical elements. Only the transfer and beam matrices and the results of various other print commands will appear in the output. The second command allows only the varied elements and the constraints to be printed. Both commands are described in section VII-D on output print control instructions.

## II-B. UNITS (Type Code 15) -- Input-Output Units

The UNITS element is almost unchanged from earlier versions of TRANSPORT. The one significant difference is that the units name is now the same as the label. It may now be placed anywhere in the element. A separate label is not permitted.

For compatability with the MAD program and for ease of performing accelerator calculations, a special command UMAD is provided. The single command UMAD causes all lengths to be expressed in meters and all angles (except for the SROT element) to be expressed in radians. The fractional momentum deviation  $\delta$  is expressed as a fraction, not as a percent. One reason these units are preferred by accelerator designers is that they are consistent. The angular measure milliradians is intrinsically self-inconsistent since it is the ratio of two lengths expressed in different units. The unit percent is self-inconsistent since a quantity in percent must be divided by 100 to be used in any equation.

The single command

UMAD

is equivalent to the series of commands

```
UNIT 1. 'M';
UNIT 2. 'R';
UNIT 5. 'M';
UNIT 6. 'N' 100.;
UNIT 7. 'R';
```

The mnemonics M' for meters and 'R' for radians are recognized by TRANSPORT and the conversion factor is supplied automatically. The mnemonic 'N' is not recognized by TRANSPORT. The units conversion containing the symbol 'N' indicates that the momentum deviation  $\delta$  is measured fractionally (not per-anything).

## II-C. SPECIAL (Type Code 16) -- Special Input Parameters

The keywords which can be used in specifying the special input parameters are

# Number Keyword Description

1. 2.	EPS FINTO	The quadratic dependence of a bending magnet field. A fringing field integral used to calculate orbit
_,		displacement.
3.	PMASS	Mass of the particles comprising the beam.
4.	HWIDTH	Horizontal half aperture of bending magnet.
5.	HGAP	Vertical half aperture of bending magnet
6.	LENGTH	Cumulative length of the system
7.	FINT	A fringing field integral used in calculating the transfer matrix.
8.	FINT2	A second integral used for the same purpose, but not very important.
12.	H1	The entrance face curvature of a bending magnet.
13.	H2	The exit face curvature of a bending magnet.
14.	RANNO	The input value of the random number used for error
		calculation. In the IBM versions of TRANSPORT if this
		element is included but left blank, the input value will be
		determined by the computer clock. It will also be printed on
		the element in the initial data listing. Its value will then
		be available for further runs with the same beam line
		configuration.
15.	FOTILT	The tilt of the focal plane.
16.	XBEGIN	The initial floor coordinate x value.
17.	YBEGIN	The initial floor coordinate y value.
18.	ZBEGIN	The initial floor coordinate z value.
19.	YAW	The plan view angle of the initial reference trajectory.
20.	PITCH	The vertical angle of the initial reference trajectory.
21.	PREF	The reference momentum to which the magnetic fields are
		normalized. If this parameter is omitted it is taken to be
		the central momentum p(0) on the BEAM element. The purpose
		of this parameter is to allow the user to change the momentum
		of the beam without changing the values of all the magnetic
		fields. The program will then automatically scale the fields
00	RMPS	to the momentum p(0) on the BEAM element.
22.	CHMI	The fractional excess bend field on a bending magnet. This
		excess field will not change the reference trajectory, but
23.	RNMS	simply steer the beam about that reference trajectory.
43.	CMMA	Overall scaling factor for the midplane-symmetry-violating
24.	VR.	field components of a bending magnet.
44.	VIL .	The vertical (midplane-symmetry-violating) bend field of a
25.	NP	bending magnet.
20.	141	The midplane-symmetry-violating normalized gradient of a bending magnet.

26. EPSP The midplane-symmetry-violating sextupole component of a

bending magnet.

27. EPS3 Cubic term for the field of a bending magnet.

An example might be the specification of the quadratic dependence of a bending magnet field. In the traditional TRANSPORT notation with type codes and positional parameters, such an element might look like

Using the mnemonic SPECIAL, it could be written as

Using keyword notation, it could also be written as

SPEC EPS = 
$$.001$$
 :

This particular special parameter can also be written directly on the element to which it applies. The BEND element will recognize the special parameter EPS. When the special parameter EPS is specified on a special parameter element, it applies to all subsequent bending magnets. When EPS is given on a bending magnet element, it applies only to that bending magnet.

II-D. ORDER (Type Code 17) -- Specification of Order of Calculation

There are no keywords associated with the order specification. In traditional TRANSPORT there are no parameters other than the type code itself. In very old versions there are additional parameters which have the suggested values of 1. 0. 3. These three numbers represent the relative magnitudes of the second, third, and fourth moments of a multidimensional Gaussian distribution for the initial beam phase space. The three numbers have long since been supplied internally by the program. They are not recognized in the input, printed in the output, nor described in the manual. The type code alone has served as the entire element.

The ORDER element has now been expanded to include two new parameters. They both represent the order of the calculation, but in different senses. If the second is omitted, it is taken as equal to the first. If the beam is always on axis, i.e. if the beam centroid always follows the reference trajectory, it is not meaningful to have separate numbers. In that case, if two numbers are given, the first is reset to equal the second.

Let us examine first the case of an on-axis beam. A single number may then be given. It specifies the order of the calculation. If this number is absent, it is assumed to equal two. The ORDER card then reverts to the traditional TRANSPORT element

17. ;

may now also be written as

ORDER ;

which is equivalent to

ORDER 2. ;

Now that some third-order matrix elements are available, it is necessary to be able to specify that a third order calculation is desired. This is done by setting this number to three. The command for a third-order run is then

ORDER 3.;

Third-order matrix elements are now included for quadrupoles (including fringing fields), sextupoles, octupoles, and combined function bending magnets. The third-order formalism for the fringing fields of bending magnets is under development, and will be incorporated into future editions.

If the beam is off axis, the transfer matrix as printed will be expanded about the off-axis central trajectory. The displacement of this off-axis central trajectory from the reference trajectory will be printed with the first order transfer matrix. The beam is displaced from the beam line reference trajectory when there is a centroid displacement (CENTROID), a known misalignment (ALIGN),

an excess horizontal bending field (Special parameter RMPS), a midplane-symmetry-violating vertical component in a horizontally bending magnet (Special parameters RNMS and VR), or a horizontally or vertically steering magnet (HKICK or VKICK). The details of how this new transfer matrix is calculated is given in section IX below.

Now two numbers are required for an order specification. The first gives the order to which the transfer matrix is calculated internally in the program. The second gives the order to which the transfer matrix is printed. When the reference point for the expansion is shifted by having the beam centroid off axis, the higher orders can contribute to the lower orders about the new reference. For example, if in the data we include the element

## ORDER 3 1;

then the first order transfer matrix will be calculated about the off-axis reference and printed. The second- and third-order matrices about the original reference will be used in calculating the off axis expansion.

This new feature brings up another possibility. It is now possible to calculate the positions of the beam centroid alone without expanding about the trajectory of the centroid. One is then calculating the progress of a single trajectory through the system. The transfer matrices may be calculated internally but not printed. In other words, if we insert the element

## ORDER 3. O. ;

then the progress of the centroid through the beam line is calculated to third order. The coordinates of that centroid are printed in the output, but no transfer matrix appears.

The element

will cause the transfer matrix to be printed to second order, but with third-order components used in calculating the off axis expansion.

To calculate the centroid position alone using first-order transfer matrices the command

ORDER 1. O. ;

is used.

## II-E. RANDOM -- Random Errors on Physical Parameters

Using a random number generator, random errors may be placed on the physical parameters used to describe a beam line. Any type of physical parameter can be given an error which falls within a specified range. The type of physical parameter and the magnitude of the error are both specified on the RANDOM. element. For example, a random error in quadrupole excitation might be represented by the element

RANDOM QUAD, B, .001;

will place a random error of maximum magnitude .001 kilogauss on the pole tip field of the quadrupoles in the system. The first keyword, in this case QUAD, gives the type of element on which the error occurs. The second gives the physical parameter to which the error is to be applied. The number gives the maximum magnitude of the error. The structure of this element is unique in that a positional parameter follows a keyword.

For an error to be applied to an element, the element specification must be written in terms of the parameters for which an error magnitude is given. If a quadrupole is specified as

QUAD L = 10., GRAD = 5.;

then the parameter B does not appear. The error specification given above in terms of B then introduces no error to the gradient of this quadrupole. An error on the gradient of a quadrupole where the gradient is given directly, as above, can be specified by

RANDOM QUAD, GRAD, .001;

At present the list of elements and their parameters to which random errors can be applied is as follows.

Element	Parameter Keywords
ROTAT DRIFT BEND	ANGLE, HGAP, FINT, H L L R PADUIS ANGLE N K1 DATE DATE WE NO
DIDITO	L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, EPS, K2, EPSP, K2P, EPS3, K3
QUAD	L, B, APERTURE, GRADIENT, K1
CENTROID	X, XP, Y, YP, L, DEL
ACCELERA	L, VOLT, LAG, FREQ
SEXTUPOL	L, B, APERTURE, K2
SOLENOID	L, B, KS
SROT	ANGLE
OCTUPOLE	L, B, APERTURE, K3

RBEND, SBEND L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, E1, E2, EPS, K2, EPSP, K2P, H1, H2, EPS3, K3, HGAP, FINT L, B, APERTURE, GRADIENT, K1

If any physical parameter is to receive a random error, then random numbers for all physical parameters are generated. If a second type of error is introduced, the random numbers used for the first type will then be unchanged. This procedure produces the type of results that a user would be likely to expect. If two types of errors are present, and the magnitude of one is reduced to zero, the result should be the same as if only the first type of error were present. An example might be an investigation of the errors of focusing elements in a system. If both quadrupoles and bending magnets are to be considered then the two error specifications

RANDOM QUAD, B, .OO1; RANDOM BEND, N, .OOO5;

could be used.

The procedure described has the disadvantage that the sequence of random numbers used for any given type of error will be changed if the beam line is changed in any way. This will be true even if the change is to an element for which no random errors are specified. An example might be the splitting up of a drift space to make two shorter drift spaces. If this is done, the beam line is considered changed. The error specification given above, where a random error is given to the magnetic fields of the quadrupoles in the system will then give a different result. The correspondence of random numbers to field values will be altered due to the generation of an additional random number for the additional drift space. It will also be possible that different versions of TRANSPORT will give different results if additional physical parameters can be given random errors. The special parameters element does not use random numbers, since it simply gives default values for the parameters of other elements.

In most cases the mathematical procedure of giving a certain error to a physical parameter is the same as the physical procedure. There are, however, two exceptions. The first is the length of a magnet. An error may be specified as

RANDOM BEND, L, 0.02;

The mathematical error, which is the procedure currently in TRANSPORT, will increase the length of a beam line when the length of magnet is increased. The physical error will correspondingly shorten the adjacent drifts so that the length of the total beam line is unchanged. It is planned to change TRANSPORT in the future so that the physical error is calculated.

The second case is the field of a bending magnet. An error may be specified by

RANDOM BEND, B, 0.005

If the field is changed, the remainder of the beam line is moved. However, in

this case, there is provision for introducing an excess field without altering the remainder of the beam line. The excess field is specified using the keyword RMPS either in the BEND element or with a special parameter. An error on this parameter may then by given by

RANDOM BEND, RMPS, 0.0001

The initial value of the random number generator may be reset using the special parameter element SPECIAL with the keyword RANNO. This initial value may also be reset by the computer clock. Detailed description is given under the element SPECIAL described in section II-C.

#### III. Input Beam Description

## III-A. BEAM (Type Code 1) -- Input Beam

The keywords which can be used in specifying the input phase space and reference momentum are:

Symbol	Keyword	Description
x	X	One-half the horizontal beam extent (cm in standard units).
x,	XP	One-half the horizontal beam divergence (mr).
у	Y	One-half the vertical beam extent (cm).
у,	YP	One-half the vertical beam divergence (mr).
l	L	One-half the longitudinal beam extent (cm).
δ	DEL	One-half the momentum spread (percent).
Po	P	The momentum of the central trajectory (GeV/c).
$\beta_{\mathbf{x}}$	BETAX	The accelerator function $\beta$ in the horizontal plane.
α <sub>x</sub>	ALPHAX	The accelerator function a in the horizontal plane
$\beta_{\mathbf{v}}$	BETAY	The accelerator function $\beta$ in the vertical plane.
$\alpha_{\mathbf{y}}$	ALPHAY	The accelerator function $\alpha$ in the vertical plane.

The same set of keywords may be used for an rms addition to the beam phase space.

The input phase space may be given by specifying its half widths or in accelerator notation. The two may not be mixed. If the half-widths and keyword notation are used the BEAM element might look like:

BEAM 
$$X=.1$$
,  $XP=1.$ ,  $Y=.1$ ,  $YP=1.$ ,  $L=0.$ ,  $DEL=0.2$ ,  $P=100.$ ;

If accelerator notation is used, the BEAM element might appear as:

BEAM BETAX=1.0, ALPHAX=.15, BETAY=2.0, ALPHAY=-.20, 
$$P = 100$$
.

The program must also be instructed to use accelerator notation with a PRINT ACCEL element. The PRINT commands are described in section VII-D. All quantities not specified on the BEAM element will be taken to be zero.

In both notations, with the exception of one special case, the central momentum must also be specified. If all the magnetic elements are described using MAD parameters, then the central momentum need not be specified. The MAD parameters use angle for bending and steering magnets, and  $K_1$ ,  $K_2$ , and  $K_3$  for the various multipole moments.

## III-B. CENTROID (Type Code 7) -- Shift in the Beam Centroid

The keywords which can be used in specifying a shift in the beam centroid are

Symbol	Keyword	Description
x	X	The shift in the x coordinate
x'	XP	The shift in x'
у	Y	The shift in y
у'	YP	The shift in y'
l	L	The shift in $\ell$ .
δ	DEL	The shift in $\delta$ .

The six parameters correspond exactly with the six positional parameters used in traditional TRANSPORT input.

The presence of a centroid shift element in the data will automatically cause the transfer matrix to be expanded about the displaced reference trajectory. Both the order used in calculating the transfer matrix, and the order to which it is expanded about the new reference trajectory are specified on the ORDER element (section II-D). There are no restrictions on the use of the ORDER card when a centroid shift is present in the data.

The CENTROID element is intended as a means of directing a particle beam into an optical system in a manner not coincident with the axis of the optical system. It would occur normally only at the beginning of a beam line. Later occurances of it would mean that the beam would incur an sudden and unexplained change in position or direction at some point in the beam line. Such usages of the centroid shift element are often to simulate the effect of misalignments or steering magnets. For such purposes, it would be better to use directly either the misalignment element (ALIGN), or one of the steering magnet elements (HKICK or VKICK).

## III-C. CORRELAT (Type Code 12) -- Rotated Beam Ellipse

The keywords which can be used in specifying the beam matrix correlations are R12, R13, R23, R14, R24, R34, R15, R25, R35, R45, R16, R26, R36, R46, R56. The keywords RIJ correspond in an obvious manner to the 15 correlations  $\mathbf{r}_1$ , among the 6 beam components. In many cases most of the correlations are zero, and it has always been a nuisance to have to type all those zeros. The use of keywords eliminates this annoyance. Now only the nonzero correlations need be typed. It is often the case that the only nonzero correlations are  $\mathbf{r}_{12}$  and  $\mathbf{r}_{34}$ . To specify the values of these correlations, the CORR element might look like

CORR R12 = 0.6, R34 = -0.8;

A user who has some sentimental attachment to the old format for the beam correlation format, or who has mostly nonzero entries, will have to make a slight revision in his data. The correlation element is the item most often requiring several lines. In previous versions of TRANSPORT an element could be continued simply by not placing a semicolon (;) at the end of a line. For compatability with the MAD program an implicit semicolon is now assumed to be in column 81 at the end of each line. A line must now be explicitly marked to be continued. This is done by placing an ampersand (&) after the data on a given line.

#### III-D. ETA -- Accelerator Eta Function

The accelerator function eta is a new element and must first be described in terms of the traditional TRANSPORT input. In some previous versions the CENTROID element was taken to be the eta function. However this identification precluded expressing the accelerator functions beta and alpha about an off-axis orbit. The centroid and the eta function are now separate. Seven items are now required for the complete specification of the eta function.

- 1 The mnemonic ETA
- (2 to 7) The coordinates x, x', y, y',  $\ell$ , and  $\delta$  defining the initial coordinates of the eta function. If the coordinate for  $\delta$  is left undefined, it is taken as one of whatever unit is being used for  $\delta$ .

The keywords which can be used in specifying the accelerator eta function are

Symbol	Keyword	Description
$\eta_{_{\mathbf{X}}}$	ETAX	The x component of the accelerator eta function.
$\eta_{\mathbf{x}}^{ ,}$	DETAX	The derivative of the x component of eta.
	ETAY	The y component of the accelerator eta function.
$\eta_{\mathbf{y}}$	DETAY	The derivative of the y component of eta.
$\eta_{\mathcal{Q}}$	ETAL	The longitudinal displacement of eta.
$\eta_{\phi}$	ETAP	The fractional momentum deviation for eta (normally 1.0)

The use of accelerator notation for the beam matrix is specified by use of the PRINT ACCEL element. It is described in section VII-D. The beam matrix will then be printed in accelerator notation and will contain the four transverse components of the eta function. They will be labelled in the output so there will be no difficulty in identifying them.

The accelerator parameters are typically expressed in meters and radians. To ensure that the eta function is expressed in such units, appropriate units changes should be made at the beginning of the data. The single command UMAD will change all units to those used most often for accelerator physics. This single command is described in section II-B.

## IV. Parameters and Algebraic Expressions

#### IV-A. PARAMETER -- Parameters

A numerical value to be used many times over in the definition of elements can be specified a single time by the use of a parameter statement. The parameter is given a proper name via the parameter element. It is then referred to by that proper name on all other elements. For example, a drift length which is to be .5 meters can be defined by

$$LD1: = .5$$
;

or alternatively

PARAM LD1 = 
$$0.5$$
;

In defining a drift space D1 whose length is LD1, one can write

D1: DRIFT, 
$$L = LD1$$
;

It is also possible to define a parameter by its use on a given element. On a later element it is represented by the label of the first element followed by the keyword of the parameter in brackets. The length of the drift space D1 may now be given directly on the drift specification as:

D1: DRIFT, 
$$L = 0.5$$
;

A second drift space, whose length is always to be the same as that of the first can be given as

D2: DRIFT, 
$$L = D1[L]$$
;

#### IV-B. Algebraic expressions

Any algebraic expression made up from previously defined parameters can also serve as a parameter. This algebraic expression can be evaluated either on another PARAMETER element, or as one of the physical parameters of a physical element. In the example above, a second drift space can be defined in terms of the parameter LD1

```
D2: DRIFT, L = 10.0 - LD1;
```

The value of the parameter LD1 may be varied, but the sum of the two drift lengths will be held fixed at 10.0 meters. The operations which may be used in the algebraic expression are:

```
Addition
       Subtraction
       Multiplication
       Division
SORT
       Square root
       Natural logarithm (written Ln in engineering notation).
ALOG
EXP
       Exponential function
SIN
       Sine function
COS
       Cosine function
SINH
       Hyperbolic sine function
COSH
       Hyperbolic cosine function
ASIN
       Inverse sine function
ACOS
       Inverse cosine function
```

The lengths of three drift spaces in a system can be made to increase in geometric progression. The first two drift spaces D1 and D2 can be made free parameters and given as:

```
D1: DRIFT, L = LD1; D2: DRIFT, L = LD2;
```

A third drift space can be defined as

```
D3: DRIFT, L = LD2*LD2/LD1
```

## V. - Physical Elements

## V-A. DRIFT (Type Code 3) -- Field free region

A single keyword is used in specifying a drift space. It is L which indicates the length of the drift space. A specification for a drift space D1 of length 100.0 meters has traditionally been written as

3. 100. 'D1';

It may now also be written as

DRIFT 100. 'D1';

or

DRIFT L = 100. 'D1';

or

D1: DRIFT L = 100.;

or in any of a number of other ways as explained in section I-A.

## V-B. BEND (Type Code 4) -- Wedge Bending Magnet (Without Fringing Fields)

The keywords which can be used in specifying a wedge bending magnet are

Symbol	Keyword	Description
l	L	The magnet length (normally meters)
В	В	The magnetic field (normally kilogauss)
ρ	RADIUS	The radius of curvature of the reference trajectory
		(normally meters).
α	ANGLE	The angle through which the reference trajectory is bend
		(normally degrees).
n	N	The normalized field derivative.
. <b>K</b> 1	K1	The quadrupole coefficient (normally meters $^{-2}$ ).
rs	RMPS	The fractional excess magnetic field.
ra	RNMS	Scaling factor for midplane-symmetry-violating field
-		components.
<b>v</b> <sub>r</sub>	VR	The midplane-symmetry-violating dipole magnetic field.
n,	NP	The midplane-symmetry-violating normalized field derivative.
K,'	K1P	The midplane-symmetry-violating quadrupole coefficient.
$\epsilon$	EPS	The quadratic field dependence.
K <sub>2</sub>	K2	The sextupole coefficient.
€'	EPSP	The midplane-symmetry-violating quadratic field dependence.
K <sub>2</sub> '	K2P	The midplane-symmetry-violating sextupole coefficient.
$\epsilon_{3}$	EPS3	The cubic field dependence.
K <sub>s</sub>	К3	The octupole coefficient.

This set of variables contains a great deal of redundacy. The bending magnet, used as an example earlier, may be described to first order in any of ten ways. We assume the beam momentum to be 600 GeV/c. The following five items are then all equivalent.

```
BEND L = 10., B = 20., N = 0.5;

BEND L = 10., ANGLE = 10., N = 0.5;

BEND L = 10., RADIUS = 1000., N = 0.5;

BEND B = 20., ANGLE = 10., N = 0.5;

BEND RADIUS = 1000., ANGLE = 10., N = 0.5;
```

Here the angle is assumed to be in milliradians. The normal units of TRANSPORT are degrees. Units other than the normal ones may be specified by use of a UNITS element (type code 15). In addition the field gradient may be specified by K1 instead of N. The two are related by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$$

The MAD program specifies a bending magnet to first order in terms of length, bend angle, and  $K_1$ .

The expansion of the magnetic field on the nominal magnetic midplane is given by

$$B_{y} = B_{o}(1+r_{s})(1 - nhx + \beta h^{2}x^{2} + \gamma h^{3}x^{3} + ...)$$

$$= B_{o}(1+r_{s}) \sum K_{n}x^{n}$$

$$B_{x} = B_{o}r_{a}(v_{r} - n'hx + \beta'h^{2} + ...)$$

The vertical component B is midplane symmetric. The quantity r represents the fractional excess field. The reference trajectory is defined as if r equalled zero. The field of the magnet may then be increased or decreased to steer the beam about the reference trajectory. If r is not specified it is set equal to the default value given on the special parameter element (type code 16.). If the default value is also unspecified, it is taken to be zero.

The sextupole component  $\beta$  is related to the coefficients  $\epsilon$  and  $K_2$  by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B\rho} \frac{\partial^2 B_y}{\partial x^2}$$

Only one of the two quantities  $\epsilon$  and  $K_2$  should be specified. If neither is specified, the sextupole component is set equal to the default value given on the special parameter element (type code 16.). If the default value is also unspecified, it is taken to be zero.

The octupole component  $\gamma$  is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B\rho} \frac{\partial^3 B_y}{\partial x^3}$$

The default value for the octupole component  $\epsilon_3$  is also set by the special parameters (type code 16.). Otherwise it is taken to be zero.

In the MAD program itself the values of  $K_2$  and  $K_3$  are defined as the normalized derivatives of the field. The value of  $K_2$  in MAD is then a factor of 2 greater than the value defined above. The MAD value of  $K_3$  is a factor of 6 (= 3!) greater than that used here.

The four parameters,  $r_a$ ,  $v_r$ , n', and  $\beta'$  may also be set on the individual bend element, or left to equal the default value. The default values may be set on the special parameter element. Otherwise they are all zero. The parameters  $K_1'$  and  $K_2'$  are related to n',  $\beta'$ , and  $\epsilon'$  respectively in the same way that  $K_1$  and  $K_2$  are related to n,  $\beta$ , and  $\epsilon$ .

The keyword TILT indicates that the bending magnet is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word TILT appears alone without a value, the angle of rotation is taken to be 90 degrees.

V-C. ROTAT (Type Code 2) -- Fringing Fields and Pole-Face Rotations for Bending Magnets

The keywords which can be used in specifying a pole face rotation are

Symbol	Keyword	Description
β	ANGLE	Angle of pole-face rotation (degrees).
g/2	HGAP	Half gap
K <sub>1</sub>	FINT	Fringe field integral
$1/R_{1,2}$	H	Pole face curvature

In traditional TRANSPORT notation, only the pole-face rotation angle is given in the ROTAT element. The other three parameters are given by the special parameter element. The keywords HGAP, FINT, and H allow the user to override temporarily the default values given by the special parameter cards. The values specified by these three keywords on any ROTAT element will apply only to that element. The default values which are assumed if not specified through the special parameter cards are given in the section describing the special parameters (type code 16).

The use of keywords to override the defaults for special parameters allows the user to specify the characteristics of the pole face rotation on the card for that element. A complete bending magnet with fringe fields and pole face rotations and curvatures might be written as

```
ROTAT .5 H = 0.01;
BEND 10. 20. 'BM';
ROTAT .5 H = -0.1;
```

# V-D. RBEND and SBEND -- Bending Magnets with Fringing Fields

The mnemonic RBEND indicates a rectangular bending magnet. The rectangular shape pertains to the entire magnet, as seen from above (plan view). The default value for both the entrance and exit pole face rotation angles is half the bend angle. The mnemonic SBEND indicates a sector bending magnet. Here the reference trajectory enters and exits from the magnet perpendicular to the magnet pole faces. The default value for both the entrance and exit pole face curvatures is zero. These mnemonics are the ones used to describe a bending magnet in the MAD program.

The default value for both the quadrupole and sextupole components of the field is zero. The default values for the entrance and exit pole face rotation angles are both zero.

A basic zero gradient RBEND or SBEND with default pole face configurations can be specified in traditional TRANSPORT notation. A sector bending magnet of length 10 meters and magnetic field of 20 kilogauss might be written as follows.

```
SBEND 10. 20. 'BS';
```

Additional parameters may be inserted by using the keyword notation. The element may also be specified entirely in keyword notation. It would then appear as

```
SBEND L = 10., B = 20. 'BS';
```

The keywords recognized for RBEND and SBEND include all those recognized for BEND and for the pole face rotation ROTAT. The SBEND defined above is equivalent to:

```
ROTAT O.;
BEND 10. 20. 'BS';
ROTAT O.;
```

The keywords used for RBEND and SBEND are

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
В	<b>B</b>	The magnetic field (normally kilogauss)
ρ	RADIUS	The radius of curvature of the reference trajectory
α	ANGLE	The angle through which the reference trajectory is bent
		(normally degrees).

n N The normalized field derivative.  $K_1 = \frac{1}{B\rho} \frac{\partial B}{\partial x}$   $K_1 = \frac{1}{B\rho} \frac{\partial B}{\partial x}$  The quadrupole coefficient (normally meters  $^{-2}$ ).

r RMPS The fractional excess magnetic field.

r RNMS Scaling factor for midplane-symmetry-violating field components.

 ${f v_r}$  VR The midplane-symmetry-violating dipole magnetic field.

n' NP The midplane-symmetry-violating normalized field derivative.

K' K1P The midplane-symmetry-violating quadrupole coefficient.

 $\beta_1$  E1 The rotation angle for the entrance pole face (default 0.0)

 $\beta_2$  E2 The rotation angle for the exit pole face (default 0.0).

 $\epsilon$  EPS The quadratic field dependence.

K<sub>2</sub> K2 The sextupole coefficient (default 0.0), defined as

$$K_2 = \frac{1}{2B\rho} \frac{\partial^2 B_y}{\partial x^2}$$

 $\epsilon$ ' EPSP The midplane-symmetry-violating quadratic field dependence.

K2 K2P The midplane-symmetry-violating sextupole coefficient.

H1 The curvature of the entrance pole face (default 0.0).

H2 The curvature of the exit pole face (default 0.0).

 $\epsilon_{\rm 3}$  EPS3 The cubic field dependence.

K<sub>3</sub> K3 The octupole coefficient (default 0.0), defined as

$$K_3 = \frac{1}{6B\rho} \frac{\partial^3 B_y}{\partial x^3}$$

g/2 HGAP The half gap of the magnet (default 0.0).

FINT The field integral (given the symbol K<sub>1</sub> in TRANSPORT).

THIT The rotation angle about the entrance reference trajectory.

In the MAD program itself the values of  $K_2$  and  $K_3$  are defined as the normalized derivatives of the field. The value of  $K_2$  in MAD is then a factor of 2 greater than the value defined above. The MAD value of  $K_3$  is a factor of 6 (= 3!) greater than that used here.

The keyword TILT indicates that the bending magnet is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word TILT appears alone without a value, the angle of rotation is taken to be 90 degrees.

A sector bend with slight pole face rotation angles of 2.0 and 3.0 degrees might then be written as

SBEND 10. 20. E1 = 2.0, E2 = 3.0 'BS';

#### V-E. QUADRUPO (Type Code 5) -- Magnetic Quadrupole

The keywords which can be used in specifying a magnetic quadrupole are

Symbol	Keyword	Description
l	L	The magnet length (normally meters).
Bo	В	The magnetic field (normally kilogauss).
a	APER (TURE)	The magnetic aperture (normally centimeters).
g	GRAD (IENT)	The gradient of the magnetic field (normally $kg/cm$ ).
K <sub>1</sub>	K1	The normalized quadrupole component.
	TILT	The angle of rotation about the optic axis.

Not all these quantities are to be used in the specification of a magnetic quadrupole. The following three elements are all equivalent.

QUAD 
$$L = 5$$
.,  $B = 10$ .,  $APER = 5$ . 'Q1';   
QUAD  $L = 5$ .,  $GRAD = 2$ . 'Q1';   
QUAD  $L = 5$ .,  $K1 = .0001$  'Q1';

Here we have assumed a beam momentum of 600 GeV/c. The description in terms of K1 has the advantage that it does not require a separate specification of the beam momentum. The quantities  $K_1$  and g are related to the pole tip field B and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_o}{a}$$

The keyword TILT indicates that the quadrupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word TILT appears alone without a value, the angle of rotation is taken to be 45 degrees. This configuration is known to accelerator designers as a skew quadrupole.

#### V-F. ALIGN (Type Code 8) -- Magnet Alignment Tolerances

The keywords which can be used in specifying magnet alignment tolerances are

Symbol	Keyword	Description
$\Delta \mathbf{x}$	X	The magnet displacement in the x direction (cm).
$\Delta \theta_{\mathbf{x}}$	RX	A rotation about the x axis (mr).
Δy	Y	The magnet displacement in the y direction (cm).
$^{\Delta heta}\mathbf{y}$	RY	A rotation about the y axis (mr).
$\Delta \mathbf{z}$	$\mathbf{Z}$	The magnet displacement in the z (beam) direction (cm).
$\Delta  heta_{f z}$	RZ	A rotation about the z axis (mr).
	CODE	A three-digit code number specifying the type of misalignment.

The seven parameters correspond exactly with the seven positional parameters used in traditional TRANSPORT input.

The meaning of the CODE digit has also been extended. Previously the hundreds digit distinguished between an uncertainty in position (OXX.) and a known displacement (1XX.). A third option is now available. It is a random displacement, denoted by the code (2XX.). The effect of the misalignment will be calculated in the same manner as if the parameters describing the misalignment were known. However, those parameters will be determined by multiplying the parameters on the misalignment by random numbers. The random numbers are distributed uniformly from -1 to 1. The parameters on the misalignment element then represent the maximum magnitude of the misalignment. The random numbers are drawn from the same sequence as those used for errors on the physical parameters of the elements. For a description of such errors, the user should look in section II-E.

# V-G. ACCELERA (Type Code 11) -- Travelling Wave Linear Accelerator

The keywords which can be used in specifying an accelerating element are

Symbo	ol Keyword	Description
L	L	Accelerator length (meters).
	VOLT	Energy gain (GeV).
φ	LAG	Phase lag (degrees).
λ	FREQ	Wavelength in cm.

The four parameters correspond exactly with the four positional parameters used in traditional TRANSPORT input.

## V-H. SEXTUPOL (Type Code 18) -- Magnetic Sextupole

The keywords which can be used in specifying a magnetic sextupole are

Symbol Keyword	Description
L L	The magnet length (normally meters).
B <sub>o</sub> B	The magnetic field (normally kilogauss).
a APER (TURE)	The magnetic aperture (normally centimeters).
K <sub>2</sub> K2	The normalized sextupole component.
TILT	The angle of rotation about the optic axis.

Not all these quantities are to be used in the specification of a magnetic sextupole. The following two elements are equivalent.

SEXT L = 5., B = 10., APER = 5. 'S1';  
SEXT L = 5., 
$$K2 = .00002$$
 'S1';

Here we have assumed a beam momentum of 600 GeV/c. The description in terms of K2 has the advantage that it does not require a separate specification of the beam momentum. The quantity  $K_2$  is related to the pole tip field  $B_0$  and the aperture a by:

$$K_2 = \frac{1}{(B\rho)} \frac{B_0}{a^2}$$

It should be noted that in the MAD program itself, the parameter  $K_2$  is defined to be the normalized second derivative of the field. The MAD value of  $K_2$  is then greater by a factor of 2 than the value defined here.

The keyword TILT indicates that the sextupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word TILT appears alone without a value, the angle of rotation is taken to be 30 degrees.

## V-I. SOLENOID (Type Code 19) -- Solenoid

The keywords which can be used in specifying a solenoid are

Symbol	Keyword	Description
L	L	Effective length of the solenoid (meters).
В	В	The field (kilogauss).
θ	KS	The solenoid strength in terms of the reference momentum
		rotation angle.

Not all these quantities are needed in the specification of a solenoid. A solenoid may be specified by its length and field or its length and reference momentum rotation angle. The relation among these three quantities is

$$\frac{B\ell}{(B\rho)} = 2\theta$$

If the reference coordinate system is rotated by the angle  $\theta$  after the solenoid, then the transfer matrix will take the block diagonalized form similar to that of a quadrupole.

## V-J. SROT (Type Code 20) -- Coordinate Rotation

A single keyword is used in specifying a coordinate rotation. The keyword is ANGLE, the angle of rotation. In traditional TRANSPORT notation, a coordinate rotation of  $90^\circ$  was represented as

20. 90. ;

It may now be represented as

SROT 90. ;

or as

SROT, ANGLE = 90;

In previous versions of TRANSPORT the transfer and beam matrices and the floor coordinates were always printed in the rotated system. It is also possible to have all these quantities printed in the unrotated system. Then the directions of vertical and horizontal can be unchanged even if the beam line contains many magnets rotated about the beam axis at various angles. The command

### PRINT REFER ;

placed at the beginning of the data, will cause the various matrices and coordinates to be printed in the unrotated system. The constraints will also pertain to the unrotated system.

The transfer matrix elements and the accelerator functions  $\beta$ ,  $\alpha$ , and  $\eta$  will now have the physical meaning one would normally expect them to have. Specifically, the transfer matrix elements will now not have mysterious gaps or regions of value zero when traversing a vertically bending magnet. These gaps occurred because the transfer matrix temporarily became block off-diagonalized when the coordinate system was rotated by 90 degrees.

## V-K. OCTUPOLE -- Magnetic Octupole

Octupole magnets are used to affect third-order aberrations in charged particle optical systems. The effect of an octupole is limited to third and higher orders. In a first- or second-order calculation, this element will act as a drift space.

Using traditional positional TRANSPORT notation, four items are needed to describe an octupole.

1 - The Mnemonic OCTU(POLE).

2 - The effective length (meters).

- 3 Field at pole tips. Both positive and negative fields are possible. A positive field is defined by the same convention as for dipoles, quadrupoles, and sextupoles. A positive field will cause a beam particle crossing the positive x axis to be deflected to the right.
- 4 The half aperture (cm).

The keywords which can be used in specifying a magnetic octupole are

Symbol Keyword

Description

L

The magnet length (normally meters).

B

B

The magnetic field (normally kilogauss).

APER(TURE)

The magnetic aperture (normally centimeters).

K<sub>3</sub>

K3

The normalized octupole component.

Not all these quantities are to be used in the specification of a magnetic octupole. The following two elements are equivalent.

The angle of rotation about the optic axis.

OCTU L = 5., B = 10., APER = 5. '01';
OCTU L = 5., K1 = .000004 '01';

Here we have assumed a beam momentum of 600 GeV/c. The description in terms of K3 has the advantage that it does not require a separate specification of the beam momentum. The quantity  $K_3$  is related to the pole tip field B and the aperture a by:

$$K_3 = \frac{1}{(B\rho)} \frac{B_0}{a^3}$$

TILT

It should be noted that in the MAD program itself, the parameter  $K_3$  is defined to be the normalized third derivative of the field. The MAD value of  $K_3$  is then greater by a factor of 6 (= 3!) than the value defined here.

The keyword TILT indicates that the octupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word TILT appears alone without a value, the angle of rotation is taken to be 22.5 degrees.

### V-L. PLASMALE -- Plasma (Lithium) Lens

A plasma lens has focusing characteristics similar to those of a quadrupole, except that the plasma lens can focus in both transverse planes simultaneously. The traditional TRANSPORT specification for a plasma lens is the same as that for a quadrupole. The keywords which can be used in specifying a plasma lens are also the same as for a quadrupole. They are here repeated for convenience.

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
В	В	The magnetic field (normally kilogauss).
a	APER (TURE)	The magnetic aperture (normally centimeters).
g	GRAD (IENT)	The gradient of the magnetic field (normally kg/cm).
K <sub>1</sub>	K1	The normalized quadrupole component.

Not all these quantities are to be used in the specification of a plasma lens. The following three elements are all equivalent.

```
PLAS L = 5., B = 10., APER = 5. 'Q1';

PLAS L = 5., GRAD = 2. 'Q1';

PLAS L = 5., K1 = .0001 'Q1';
```

Here we have assumed a beam momentum of 600 GeV/c. The description in terms of K1 has the advantage that it does not require a separate specification of the beam momentum. The quantities  $K_1$  and g are related to the pole tip field  $B_0$  and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_0}{a}$$

V-M. HKICK or VKICK (Type Codes 35 and 36) -- Horizontal and vertical steering magnets

Using traditional positional TRANSPORT input, the description of HKICK and VKICK is much the same as that of a bending magnet. There are three items needed.

- 1 The mnemonic HKICK or VKICK
- 2 Effective length (meters).
- 3 The central field strength (kilogauss).

A positive angle bends to the right for the HKICK element or down for the VKICK element. There is no provision for non-uniform field in this element. The fringe fields are included automatically.

The difference between a steering magnet and a bending magnet is that the steering magnet does not affect the reference trajectory. If the beam line is perfectly aligned with no errors, and the beam centroid follows the reference trajectory, the field of the steering magnets is zero. The steering magnets are used to compensate for errors and misalignments in the beam line. If they are given a nonzero field, the beam line reference trajectory is unaffected, but the beam is steered toward the reference trajectory.

The keywords which can be used in specifying steering magnets are seems

Symbol	Keyword	Description
l	L	The effective length of the magnet (meters).
<b>B</b> .	В	The magnetic field (kilogauss).
α	ANGLE	The deflection angle (deg).
	TILT	The angle of rotation about the optic axis.

Not all these quantities are to be used in the specification of a steering magnet. Assuming a 600 GeV beam and angles measured in milliradians, the following two element are equivalent

```
HKICK L = 1.0, B = 2.0;
HKICK L = 1.0, ANGLE = 0.1;
```

The description in terms of angle is used by the program MAD. However, the program MAD currently uses the keyword KICK instead of ANGLE. At this point, the difference in keyword is useful, since the conventions for the sense of the angle are opposite. TRANSPORT uses the same convention for a steering magnet as for a bending magnet.

The elements HKICK and VKICK should always be used for steering magnets. The centroid shift element CENTROID should never be used. The steering magnet elements contain a complete set of first-, second-, and third-order transfer matrix elements. This includes the effect of having the angle of deflection vary inversely with the momentum of the particle. The centroid shift element contains none of these effects, but simply deflects the beam in a momentum independent way.

### VI. Beam Lines

## VI-A. REPEAT (Type Code 9) -- Repetition

The REPEAT command is used in much the same manner as in traditional TRANSPORT input. The type code 9 may now be replaced by the mnemonic REPEAT. It is followed by a single number. If that number is positive the REPEAT element indicates the beginning of a section to be repeated. The number gives the total number of times that section is to be included consecutively in the beam line. If that number is zero, the REPEAT element indicates the end of the section to be repeated.

An example would be much the same as one using traditional TRANSPORT notation.

REPEAT 9. ;

(section to be repeated 9 times)

REPEAT O. ;

## VI-B. SECTION (Type Code 24) -- Defined Section

The keywords which can be used in defining a section of beam line are BEGIN, END, FORWARD, and BACKWARD. If a section of a beam line is to be identified to be used again at some later point, it is preceded by the command

```
SECTION BEGIN 'FRED';
```

and followed by the command

SECTION END 'FRED';

The label, in this case 'FRED', identifies the section for later reference. If it is desired to repeat the same section at some later point in the beam line, one uses the command

```
SECTION FORWARD 'FRED' :
```

If the section is to be repeated but in reverse order, use the command

```
SECTION BACKWARD 'FRED';
```

The difference between the REPEAT command and the SECTION command is that the SECTION command does not require that the repetitions immediately follow the original. They may be separated by other portions of beam line. Thus we have that the sequence

```
REPEAT 5.; (section of beam line)
REPEAT O.;
```

is identical to the sequence

```
SECTION BEGIN 'SAM';
(section of beam line)
SECTION END 'SAM';
SECTION FORWARD 'SAM';
SECTION FORWARD 'SAM';
SECTION FORWARD 'SAM';
SECTION FORWARD 'SAM';
```

However, the example

```
SECTION BEGIN 'SAM';
(section of beam line)
SECTION END 'SAM';
(other elements)
SECTION FORWARD 'SAM';
```

cannot be done with the REPEAT element.

#### VI-C. Assembling Beam Lines with MAD Notation

Simple Beam Lines

Using MAD notation, a simple beam line to be studied may be defined as a sequence of elements. As an example we define a simple beam line and give it the label 'SIMP'. It is composed from previously defined drifts D1 and D2, quadrupoles Q1 and Q2, and bending magnet B1.

SIMP: LINE = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)

The alternative notation, used in earlier versions of MAD, will be equivalent

LINE SIMP = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)

Sub-Lines

Lines themselves may serve as elements of other lines. Such a sub-line must be defined in a separate statement. Thus one can easily include a subline several times in a given beam line. Defining two such sublines, the above example may be written as

DOUB: LINE = (D1, Q1, D2, Q2)

BPD: LINE = (B1,D2)

SIMP: LINE = (DOUB, D2, BPD, BPD, Q2, D2, Q1, D1)

Repetition and Reflection

A repeated element can be indicated by preceding it with an integer followed by an asterisk Thus our line SIMP becomes

SIMP: LINE = (DOUB, D2, 3\*BPD, Q2, D2, Q1, D1)

Several consecutive elements may also be enclosed in parentheses and repeated. We could forgo the definition of BPD and write our beamline as

SIMP: LINE = (DOUB, D2, 3\*(B1, D2), Q2, D2, Q1, D1)

A subline which is to be reflected (reverse order of elements) is indicated by preceding it with a minus sign. Thus our beam line SIMP may finally be shortened to

SIMP: LINE = (DOUB, D2, 3\*(BPD), -DOUB)

In the TRANSPORT implementation of MAD notation, a repetition indication may not be preceded with a negative sign. Nothing is lost by this restriction since a reflected repeated sub-line is the same as a repeated subline with its elements reversed and in reverse order.

## Replacable Arguments

A beam line definition may include formal arguments. Our sub-line DOUB, for example, may not have Q2 included explicitly, but have a formal argument in its place. It is then defined as

```
DOUB: LINE(X) = (D1,Q1,D2,X)
```

The formal arguments are included in the sequence of elements as if they were labels representing physical elements or sub-lines. They must also be included in parentheses before the equal sign.

When the sub-line DOUB is used, the formal argument must be replaced by the actual argument in parentheses following the label DOUB. Thus the line SIMP becomes

```
SIMP: LINE = (DOUB(Q2), D2, 3*(BPD), -DOUB(Q2))
```

Formal arguments may be single elements or sub-lines. When a subline has a single formal argument, it must be used with a single actual argument. When the subline has two formal arguments, it must be used with two actual arguments. In all cases, the number of formal arguments must be the same as the number of actual arguments.

### USE Statement

The USE statement specifies which of the defined beam lines and sublines is to be used in the calculation. In our example, the most obvious candidate is the complete assembled beam line SIMP. The USE command would then read

```
USE SIMP ;
```

If, however, we wished to investigate the properties of the entrance doublet without creating a new data deck, then we could simply use the command

```
USE DOUB;
```

TRANSPORT would then run through only the beamline named DOUB.

It is the USE statement which differentiates between the MAD procedure of defining beam lines explicitly and the traditional TRANSPORT procedure of just listing the elements in order. If no USE statement is included in the data, TRANSPORT considers the beam line to consist of the elements in the order listed.

## VI-D. MARKER -- Location markers

A marker has no additional keywords but is completely given by the word MARKER plus a label. It is used to identify a given location in a beam line. The marker element allows the definition of a beam line to be uninterrupted by the specification of operations. It can be composed strictly of a sequence of physical elements, such as magnets and drift spaces. Any print statements, constraints, matrix updates or other operations may be defined subsequent to the definition of the beam line. They may refer to the markers placed in the beam line. When TRANSPORT runs through the beam line, it executes the operations when it encounters the corresponding marker.

We can place two markers in the example used in the previous pages. Defining the markers as MAR1 and MAR2 involves adding two elements

```
MAR1: MARKER; MARKER;
```

These markers may then be inserted into the beamline SIMP

```
SIMP: LINE = (DOUB, D2, 3*(BPD), MAR1, -DOUB, MAR2)
```

Reference to the markers can be made by any of the elements specifying operations described in section VII.

## VII - Operations

## VII-A. UPDATE (Type Code 6) -- Transfer Matrix Update

There are two keywords recognized by the UPDATE element. They are used without values giving the element the appearance of a simple instruction. To update the transfer matrix R, use the element.

UPDATE R ;

To update the auxilliary transfer matrix R2, one uses the element

UPDATE R2;

If the beam line is assembled from elements and specified in a LINE statement, the UPDATE command may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

MARKER 'MAR1';

is included in the beam line description. Assume further that the user wishes to update the transfer matrix at the location of this marker. The command

UPDATE R, LOCA = MAR1;

will perform the desired update operation.

An update may be used no matter what the order of the run. However, the beam matrix will be unaffected by the update only if the order of the transfer matrix being printed is no greater than first. This order is the second number on the ORDER element. In second and higher orders the aberrations cause the ellipse to become distorted. The subsequent distribution is described in terms of its first and second moments. An update causes the program to assume the distribution at the update is an ellipsoid with the given first and second moments. The subsequent behavior of this ellipsoid may differ from what would occur without this refitting procedure.

### VII-B. FIT (Type Code 10) -- Fitting Constraints

The keywords which can be used in specifying a fitting constraint are

Keyword Description

I First code digit indicating matrix element.

J Second (set of) code digit(s) indicating matrix element.

VALUE Desired value of matrix element.

TOLERANC Desired accuracy of fit (standard deviation).

NAME Name of matrix element to be constrained.

The first four parameters correspond to the positional parameters used in traditional TRANSPORT input. The keyword NAME is used as an alternative to the two code digits I and J. In traditional TRANSPORT notation a constraint of the  $R_{1.2}$  transfer matrix element to zero appears as

Using keyword notation, this same constraint may be rewritten as

FIT 
$$I = -1.$$
,  $J = 2.$ ,  $VALUE = 0.0$ ,  $TOLER = 0.001$ ;

If the NAME keyword is used to specify the matrix element, then the same constraint would be

FIT NAME = 
$$R12$$
, VALUE = 0.0, TOLER = 0.001;

The positional parameter notation is convenient because it does not require the user repeatedly to type in the names of the keywords. For this reason, a mixed notation may be the most desirable. The same constraint can be represented as

FIT 0. 0. 0. 0.001 NAME = 
$$R12$$
;

The traditional positional TRANSPORT notation is used, leaving zeros for the two code digits. This is necessary since the desired value and the tolerance of the fit are defined as the third and fourth positional parameters respectively. The matrix element specification after the keyword NAME then overrides the two zeros and the fit is made to R12.

A complete list of matrix elements which may be identified by keyword follows. The lower-case letters i, j, k, and L stand for integers which specify the particular matrix element. The TRANSPORT output for the fit element is printed using the same symbols.

```
Symbol |
          Matrix Element
          An element of the first-order transfer matrix R.
Rij
          An element of the auxilliary first-order transfer matrix R2.
RAij
          An element of the second-order transfer matrix T.
Tijk
          An element if the auxilliary second-order transfer matrix T2.
TAijk
          An element of the third-order transfer matrix U.
Uijkl
UAijke
          An element of the auxilliary third-order transfer matrix U2.
Sij
          An element of the beam matrix \sigma.
Cij
          An element of the beam correlation matrix r.
XC
          The beam centroid position in x.
XPC
          The beam centroid position in x'.
YC
          The beam centroid position in y.
YPC .
        The beam centroid position in y'.
DLC
          The beam centroid position in \ell.
DELC
          The beam centroid position in \delta.
L
          The beam line accumulated length.
XFLOOR
          The floor coordinate x value.
YFLOOR
          The floor coordinate y value.
ZFLOOR
          The floor coordinate z value.
YAW
          The floor coordinate plan view reference trajectory angle.
PITCH
          The angle the reference trajectory makes with the horizontal.
ROLL
          The net rotation angle about the reference trajectory.
BETAX
          The accelerator function \beta_{\mathbf{v}}.
ALPHAX
          The accelerator function \alpha_{\downarrow}.
BETAY
          The accelerator function \beta_{v}.
ALPHAY
          The accelerator function \alpha_{\mathbf{v}}.
          The accelerator function \eta_{_{\mathbf{Y}}}.
ETAX
DETAX
          The accelerator function \eta_{\mathbf{v}}.
ETAY
          The accelerator function \eta_{v}.
DETAY
          The accelerator function \eta_{\mathbf{v}}.
```

In addition any algebraic combination of matrix elements may also be constrained. The algebraic combination is written, as in a FORTRAN statement, after the keyword NAME. For example, a fit of the tilt of the focal plane angle, written in mixed notation, might be

```
FIT 0. 0. .3 0.001 NAME = R16/(R11*T126)
```

A number of FORTRAN supplied intrinsic functions may also be used in an algebraic expression. The intrinsic functions now available are

SQRT Square root

ALOG Natural logarithm (written Ln in engineering notation).

EXP Exponential function

SIN Sine function COS Cosine function

SINH Hyperbolic sine function
COSH Hyperbolic cosine function

ASIN Inverse sine function ACOS Inverse cosine function

Additional functions can easily be added if there is use for any of them. The element type codes 22. and 23., described in Fermilab internal report TM-1064, are no longer operative. They may be regarded as an intermediate step to the development of an algebraic notation. These two type codes are now used internally in TRANSPORT in decoding an algebraic expression. There was no way to maintain compatability and continue to allow type codes 22. and 23. to be directly input by the user. The new notation is so much clearer that no loss is user capability is anticipated.

If the beam line is assembled from elements and specified in a LINE statement, the FIT instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

### MARKER 'MAR1';

is included in the beam line description. Assume further that the user wishes to fit an element of the transfer matrix at the location of this marker. The command

FIT O O O.O O.OO1 NAME = R12, LOCA = MAR1 ;

will constrain the matrix  $R_{1\,2}$  to equal 0.0 at the location given by the marker MAR1.

## VII-C. Varying of Physical Elements

Parameters expressed in keyword notation may be varied in order to fit certain constraints. There are two ways to indicate that a parameter is to be varied:

- (1) The traditional vary code.
- (2) An explicit VARY command.

The vary code is used in the same manner as in traditional TRANSPORT. In traditional TRANSPORT a quadrupole whose field is to be varied might be specified as

The replacement of the type code by the mnemonic QUAD allows the vary code to be used in the same manner. Our quadrupole would now be specified as

The use of keyword notation continues to allow the use of the vary code. A quadrupole to be varied expressed in keyword notation might be

QUAD.01 
$$L = 10.$$
,  $B = 5.$ ,  $APER = 2.54$  'Q1';

Alternate sets of variables still permit the keyword notation. A quadrupole can be expressed in terms of its gradient instead of pole tip field and aperture. If the gradient is to be varied, the quadrupole can be specified as

QUAD.01 
$$L = 10.$$
,  $GRAD = 2.0$  'Q1';

Vary codes may be coupled as before by using integers 2-9 or letters A-Z. They may also be inversely coupled by preceding the vary code with a minus sign.

Since the concept of the vary code is positional, there is one catch when using it with a keyword specification. A vary code in a given position corresponds to a keyword parameter in the same position if the keyword parameters are ordered according to the listings in this document. In other words, if our quadrupole is written as

QUAD.01 B = 5., L = 10., APER = 
$$2.54$$
 'Q1';

the vary code will be taken to mean that the magnetic field is to be varied.

In the listing of keywords for the quadrupole, the keyword L comes before the keyword B. If the keywords in the quadrupole specification above are reordered according to the keyword listing, then B will be the second parameter. The vary code in the second position then indicates that B is to be varied.

The explicit vary command is more suited to keyword notation The above quadrupole can be specified without the vary code.

```
QUAD L = 10., B = 5., APER = 2.54 'Q1';
```

The magnetic field can be made to be variable by the statement

```
VARY, NAME = Q1[B];
```

The keyword NAME is followed by a designation of the parameter to be varied. In the case of an element parameter this designation consists of the element label followed by the parameter keyword in brackets.

Parameters declared on PARAMETER elements may also be varied. In this case the item following the keyword NAME is the parameter name. If the value of the magnetic field were declared in a separate parameter statement, the quadrupole specification might look like.

```
PARAM, B1 = 5.0;
QUAD L = 10.0, B \neq B1, APER = 2.54 'Q1';
```

The parameter B1 could be varied by means of the statement

```
VARY, NAME = B1;
```

There is no provision for coupled variation as there is with the vary codes. However, the use of parameters and algebraic expressions allows much greater flexibility than does vary codes.

The use of the VARY statement also allows separation of the mathematical operations to be performed from the beam line specification. The VARY statements can all be placed together, near the other operations, after the beam line specification is complete. This grouping of VARY statements is possible both in the case when the beam line is defined explicitly with LINE and USE statements and in the traditional TRANSPORT situation when the beam line is simply the sequence of elements listed.

A variable parameter may be fixed again by the command

```
FIX, NAME = name ;
```

Whatever parameters are named are then held fixed and are not available for achieving the desired constraints.

# VII-D. PRINT (Type Code 13) -- Output Print Control Instructions

The keywords which can be used in specifying print control instructions are

Code	77	
Digit	Keyword	Description
1	BEAM	The beam (sigma) matrix
4	TRANS	The transfer matrix R (and possibly T and U)
7	ACCELERA	The use of accelerator notation
8	ALIGN	The misalignment effect table
12	COORDINA	The floor coordinates of the reference trajectory.
13	REFER	Refer the transfer and beam matrices to the unrotated
		coordinate system.
14	WAIST	The positions and sizes of the nearby waists
16	PRECISE	Precise values of varied parameters
17	NOPARA	The suppression of physical parameters when printing
18	ONLY	The printing only of varied elements and constraints
19	ONELINE	The printing of transfer and beam matrices on a single line.
24	TRAN2	The auxilliary transfer matrix R2.
	ELEMENTS	The listing of the physical elements.
	ON	Certain print instructions are to be executed after every
		physical elemen.
	OFF	Turns off the automatic print after each element.

Most of these instructions have been already explained, either in the TRANSPORT manual or in the Fermilab report TM-1064. For example, the single transfer matrix print instruction

13. 4. ;

may now be replaced by

PRINT TRANS ;

The single beam matrix print instruction

13. 1. ;

may now be replaced by

PRINT BEAM;

Several keywords may now be placed on a single like with the single mnemonic PRINT. An example might be

PRINT BEAM, TRANS ;

This single instruction causes both the beam (sigma) matrix and the transfer matrix to be printed at the specified location. If BEAM and TRANS are included

in the same element the beam matrix will be printed before the transfer matrix, regardless of the order of the keywords.

If it is desired to have the transfer matrix printed after each physical element, then one uses the instruction

PRINT TRANS, ON ;

To terminate the automatic printing of the transfer matrix, use the command

PRINT TRANS, OFF ;

If complete suppression of the printing for a portion of the beam line is desired, one can use the element

PRINT ELEMENTS, OFF;

The printing of elements is restored by

PRINT ELEMENTS, ON ;

If the keyword ON appears in a given PRINT element, it applies to all other keywords in that element where meaningful. For example the line

PRINT ELEMENTS, ON, TRANS ;

will not only turn on the printing of elements, but also the printing of the transfer matrix after each element.

If the beam line is assembled from elements and specified in a LINE statement, the PRINT instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

MARKER 'MAR1';

is included in the beam line description. Assume further that the user wishes to fit an element of the transfer matrix at the location of this marker. The command

PRINT TRANS, LOCA = MAR1;

will print the transfer matrix to the order specified on the ORDER element at the location MAR1.

### VII-E. STORE -- Storage of Matrix Elements

Any matrix element or quantity that can be constrained can also be stored for use in a subsequent constraint. The stored quantity is given its own proper name on the STORE command. The quantity is referred to by this proper name at any later point. For example, the initial value of the synchrotron function BETAX may be stored under the name BXO.

STORE, BXO = BETAX;

If a constraint is to be imposed at the end of the beam line, setting the final beta value equal to the initial beta value, the user could impose the constraint.

FIT 0. 0. 0.0 0.001, NAME = BETAX - BXO ;

The difference BETAX - BXO would then be fit to zero. The quantity BETAX would be the horizontal value of beta at the point of the FIT element. The quantity BXO would be the initial horizontal value of beta.

If the beam line is assembled from elements and specified in a LINE statement, the STORE instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

MARKER 'MAR1';

is included in the beam line description. Assume further that the user wishes to fit an element of the transfer matrix at the location of this marker. The command

STORE BXN = BETAX, LOCA = MAR1;

will store the horizontal beta value under the name BXN at the location MAR1.

#### VIII. Third Order

### VIII-A. Order Specification

Without further instruction TRANSPORT calculates the transfer matrix to first order. The order specification allows the order of the calculation to be changed to zeroth, second, or third. Calculating to zeroth order simply means tracing the beam centroid through the system. The linearization about the centroid, or transfer matrix, is not calculated. The tracing of the beam centroid has meaning only if it is not coincident with the reference trajectory. Several mechanisms can cause the centroid to deviate from the reference trajectory. They are described below in section IX-A.

If the beam is off axis, the order of matrix which is calculated may differ from that printed. A second order transfer matrix can contribute to a first-order off-axis expansion. For example, the off-axis traversing of a quadrupole can give a bending magnet component and therefore cause dispersion. The order of the calculation must be at least as great at that displayed. The exact specification is given in section II-C above. The mathematical details of calculating the off-axis expansion are given in section IX below.

The effect of the third order transfer matrix on the beam ellipse has been incorporated into the program. The dimensions of the initial ellipse are assumed to represent the moments of a Gaussian distribution. In calculating the effect of the beam line on the phase ellipse, the sensitivity to assumptions about the initial distribution becomes greater with each additional order. The dimensions of a third-order beam ellipsoid should not be taken as being physically accurate. Rather they should be regarded as an order-of-magnitude estimate of the combined effect of all the aberrations. The dimensions of the beam ellipsoid can be used to assess whether a third-order correction scheme is necessary. Details of the phase-space distribution should be determined with the computer program TURTLE.

#### VIII-B. Third-Order Transfer Matrix Constraints

The third-order transfer matrix constraint has a form very similar to the second-order constraint. The third-order matrix element  $U_{ijk\ell}$  has four indices, i, j, k, and  $\ell$ . The first of the two index codes on the constraint element is the negative of the first index i. The second index code is made by mushing together the last three indices. To fit the third-order matrix element  $U_{ijk\ell}$  to zero, one would use the FIT element

This specification is the obvious extension of the notation already in existence for the fitting of second-order matrix elements. To constrain the second-order matrix element  $T_{ijk}$  to zero, one would use the instruction

$$FIT -i. jk. 0.0 0.001$$
;

The keyword description of a constraint on a third-order matrix element is given in section VII-B above.

VIII-C. Octupoles and Octupole Components of Bending Magnets.

The octupole magnet and octupole component of a bending magnet affect only third- and higher-order matrix elements. They are used to correct aberrations of third and higher order. Both may be varied in a third-order run to fit the third-order transfer matrix elements. The octupole element is described in section V-K above. The octupole coefficient  $\gamma$  of a bending is described in section V-B on bending magnets and also section II-B on special parameters.

## IX. Off-Axis Expansion of Matrix Elements

Several mechanisms can steer the beam off axis. The beam centroid will then no longer coincide with the reference trajectory. An off axis beam can be achieved in several ways.

- (1) A deliberate displacement can be inserted via a CENTROID element.
- (2) The beam centroid may be displaced by the misalignment of a magnet or section of beam line.
- (3) The beam may be steered off axis by an excess field on a bending magnet.
- (4) The beam may be steered off axis by a midplane-symmetry-violating vertically bending component of a bending magnet.
- (5) The beam may be deliberately steered by the use of steering magnets HKICK and VKICK.

The transformation through a single element can be represented in the same manner as in on-axis configurations, except that there is now a zeroth order term. This new term represents the displacement of the six coordinates of a ray that lies on the reference trajectory as it enters the magnetic element. The net transformation can be represented to third order by

$$X_1 = X_1 + RX_0 + TX_0X_0 + UX_0X_0X_0$$
 (IX-1)

The net effect of the magnetic system on neighboring trajectories can be represented by a first-, second-, or third-order transformation. We start with equation (IX-1) for the transformation of an arbitrary trajectory. If we reexpress the coordinates relative to the transformed original reference trajectory  $X_r$ , and denote the difference by  $\Delta X_r$ , then we have

$$X_{1}_{r} + \Delta X_{1} = X_{1}_{s} + R(X_{or} + \Delta X_{o}) + T(X_{or} + \Delta X_{o}) (X_{or} + \Delta X_{o})$$

$$+ U(X_{or} + \Delta X_{o}) (X_{or} + \Delta X_{o}) (X_{or} + \Delta X_{o})$$

$$(IX-2)$$

Subtracting the equation for the reference trajectory, we derive

$$\Delta X_{1} = R\Delta X_{o} + T(2X_{or}\Delta X_{o} + \Delta X_{o}\Delta X_{o})$$

$$+ U(3X_{or}X_{or}\Delta X_{o} + 3X_{o}\Delta X_{o}\Delta X_{o} + \Delta X_{o}\Delta X_{o}\Delta X_{o})$$

$$= (R + 2TX_{or} + 3UX_{or}X_{or})\Delta X_{o}$$

$$+ (T + 3UX_{or})\Delta X_{o}\Delta X_{o} + U\Delta X_{o}\Delta X_{o}\Delta X_{o}$$

$$(IX-3)$$

From equation (IX-3), we can define new first- and second-order transfer matrices by

$$R* = R + 2TX_{or} + 3UX_{or}X_{or}$$

$$T* = T + 3UX_{or}$$
(IX-4)

These redefined matrices for each element can then be accumulated to produce transfer matrices for the entire magnetic optical system. The transformation of a particle trajectory through the system can now be represented by a transformation similar in appearance to equation (IX-1).

$$X_1 = X_{1r} + R(t)X_0 + T(t)X_0X_0 + U(t)X_0X_0X_0$$
 (IX-5)

Here the matrices R(t), T(t), and U(t) are calculated as products of the matrices R\*, T\*, and U, as defined in equation (IX-4). They are expressed relative to the transformed original reference trajectory  $X_{1}$ . The transformed original reference trajectory is determined from equation (1) on an element-by-element basis.

Since the fractional deviation from the reference momentum is one of the trajectory coordinates, the off-axis expansion can be used to explore chromatic effects. If a nonzero of is specified on a CENTROID element, the first-order transfer matrix will be for an off momentum central trajectory. If there is dispersion in the system, this new central trajectory will, at some point, be displaced from the reference trajectory. The result from an off-axis expansion can differ substantially from that of an on-axis calculation. The effect of many orders higher than second or even third can be seen.

The most straightforward example is that of a very long focusing system, with many intermediate foci. For a slight deviation from the reference momentum, the final beam spot size will grow linearly with the size of this deviation. As the momentum is increased, however, one of the intermediate foci will move downstream and the system as a whole will once again be focusing.

An on-axis expansion to second order will show only the linear growth in spot size, no matter how great the momentum deviation. The third-order on-axis expansion will show very little difference from the second. The off-axis expansion will show the system once again coming to a focus as the momentum is increased. Even for very long systems, a third-order off-axis chromatic calculation will be indistingushable from a ray-tracing calculation using

unexpanded momentum dependence in the equations of motion through the magnetic fields.

The beam centroid displacement is printed next to the first-order transfer matrix when the beam is off axis. For a zeroth-order calculation the centroid displacement is printed alone. For a first- or lower-order calculation the same vector of six quantities will appear as the leftmost column in the beam matrix representation. For second- and third-order calculations, the column in the beam matrix may differ slightly from the column given with the transfer matrix. That is because the numbers given with the beam matrix represent the position of the center of gravity of the beam. Second- and higher-order effects introduce distortions into the beam ellipse, so that it is no longer an ellipse. The center of gravity at a given beam location it is no longer the optical image of the center of gravity at the beginning of the system. Steering magnets should always be set so that the centroid (not the center of gravity) is relaigned along the beam axis. Higher-order multipoles should then be used to make the center of gravity coincide with the centroid.

### X - Violations of Midplane Symmetry

Individual magnetic elements are typically designed to have midplane symmetry. Errors of fabrication and misalignments can cause the midplanes of the elements not to coincide. The beam line as a whole may then not have midplane symmetry. For a magnet which corresponds to a pure multipole, such as a zero gradient bending magnet (dipole), quadrupole, sextupole, or octupole, the required procedure is straightforward. The departure of the midplane symmetry from that of the beam line can be represented by the rotation and misalignment elements. The combined function bending magnet however, contains several multipole components in the same magnet. These different multipoles may have different planes of symmetry. The bending magnet also has a midplane defined by the geometry of the beam. This geometric midplane may or may not coincide with the plane of symmetry of even the dipole component.

We therefore need to introduce midplane-symmetry-violating dipole, quadrupole, and sextupole components of the field. The dipole component will cause the beam centroid to deviate from the geometrically defined reference trajectory.

The representation of these skew components is given in section V-B for bending magnets. The consequences of the centroid being diverted off axis are given in section IX.